i. Reflexivity.

$$\alpha \xrightarrow{A_2} X \to X$$

$$\alpha \to X,\ y \subseteq \alpha \xrightarrow{A_6} \cancel{X} \to y$$

ii. Augmentation

$$XZ \xRightarrow{A_2} XZ \to XZ$$

$$XZ \to XZ,\ X \to y \xRightarrow{A_3} XZ \to XZY$$

$$XZ \to XZY \xRightarrow{A_1} XZ \to YZ$$

iii. Transivity:

$$X \xRightarrow{A_2} X \to \alpha$$

$$X \to X,\ X \to Y \xRightarrow{A_3} X \to \alpha Y$$

$$X \to XY,\ Y \to Z \xRightarrow{A_3} X \to XYZ$$

$$X \to XYZ \xRightarrow{A_6} X \to Z$$

- Decomposition.

$$Z \subseteq Y \xRightarrow{A_1} Y \to Z$$

$$X \to Y, Y \to Z \xRightarrow{A_3} X \to Z$$

ii) Reflexivity.

$$x \subseteq x \xRightarrow{A_1} X \to x$$

iii) Accumulation:

$$Z \subseteq YZ \xRightarrow{A_1} YZ \to Z$$

$$X \to YZ, YZ \to Z \xRightarrow{A_3} X \to Z$$

$$X \to Z, Z \to AW \xRightarrow{A_3} X \to AW \xRightarrow{A_1, A_3} X \to A$$

$$X \to A \xRightarrow{A_2} YZX \to YZA, \quad X \to YZ \xRightarrow{A_2} X \to YZX$$

$$X \to YZX, YZX \to YZA \xRightarrow{A_3} X \to YZA$$

2a)

$X \to A \in \tilde{F}^+,\ A \notin X$

implicates $X \to V \in \tilde{F}^+$

$A \in X^+ \setminus X.\ Y \to A \in \tilde{F}$

non-trivial FD

$Y \subseteq X^+$, thus $X \to Y \in \tilde{F}^+$

$X \to Y \in \tilde{F}^+$, X SKey, thus $X \to V \in \tilde{F}^+$

2c)

R in 3NF, K only one primary key of R.

$X \rightarrow A$ FD in $\mathcal{F}$

$(K - A) \cup X$ is a superkey

$K' \subseteq (K - A) \cup X \Rightarrow K = K'$

Therefore $A \in K'$, and thus $A \in X$

FD $X \rightarrow A$ trivial $\square$

2d) K only primary key.

$V \setminus Z_i$ a super key, $1 \le i \le p$.

So $K \subseteq \bigcap_{i=1}^{p} (V \setminus Z_i) = V \setminus Z_1 \ldots Z_p$.

$\Rightarrow V \setminus Z_1 \ldots Z_p$ is superkey

$K = V \setminus Z_1 \ldots Z_p$

L a superkey, $A \in K \setminus L$, thus $A \notin L^+$

$\square$

$V, X \subseteq V$

$|F| = 2n+1$

$|\pi[X]F| = 2^n$

$V = \{A_1, \dots, A_n, B_1, \dots, B_n, C_1, \dots, C_n, D\}$

$\mathcal{F} = \{A_i \to C_i, B_i \to C_i \mid 1 \leq i \leq n\} \cup \{C_1 \dots C_n \to D\}$

$X = A_1 \dots A_n B_1 \dots B_n D$

$F^+ \ni A_1 \dots A_n \to D$

$B_1 \dots B_n \to D$

$w \in \{A_1, B_1\} \times \dots \times \{A_n, B_n\} \to D$

Sample instance: $n = 2/3$

$|\{A_1, B_1\} \times \{A_2, B_2\}| = 4$

$2^{\cancel{2}\,3}$

$\Rightarrow$ General case: 2^n

$\mathcal{F}' = \{w \to D \mid w \in \{A_1, B_1\} \times \dots \times \{A_n, B_n\}\}$

$\mathcal{F}' \subseteq \pi[X]\mathcal{F}$

$\Rightarrow |\pi[X]\mathcal{F}| = O(2^n)$

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$X \to A \in \mathcal{F}$

$R_1 = \left(V-A,\ \pi_{[V-A]}\mathcal{F}\right)$

$R_2 = \left(X.A,\ \pi_{[XA]}\mathcal{F}\right)$

4a

r_1

A	B	C
a_1	b_1	c_2
a_1	b_2	c_1
a_2	b_1	c_1
a_1	b_1	c_1

$A \not\rightarrow B$

$A \not\rightarrow C$

4b

$$F^+ = \{ A \to B,\ B \to C,\ AB \to C,\ A \to C,\ AC \to B \}$$

r_2		
A	B	C
a_2	b_1	c_1
a_1	b_1	c_1
a_3	b_2	c_1

4c)

$$\phi \Rightarrow \mu(A) = \nu(A)$$

A	B	C
a_1	*	c_1
a_1	*	c_1
a_1	*	c_1

r_3

A	B	C
a_1	b_1	c_1
a_2	b_2	c_2

4. d

$$\mathcal{F} = \{A \not\to \emptyset,\ C \not\to \emptyset\}$$

r_4

A	B	C
a_1	b_1	c_1
a_2	b_2	c_2
a_3	b_3	c_3